① (a) order 3

(b) linear

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② (a)

$f(x) = cx^4$

$f'(x) = 4cx^3$

$$xf'(x) - 4f(x) = x(4cx^3) - 4(cx^4)$$
$$= 4cx^4 - 4cx^4$$
$$= 0$$

(b) $f(x) = cx^4$ solves $xy' - 4y = 0$.

Need $f(2) = 4$.

Plug in to get: $4 = c(2)^4$

$$4 = 16c$$
$$c = \frac{1}{4}$$

Thus, $\boxed{f(x) = \frac{1}{4}x^4}$ solves the initial value problem.

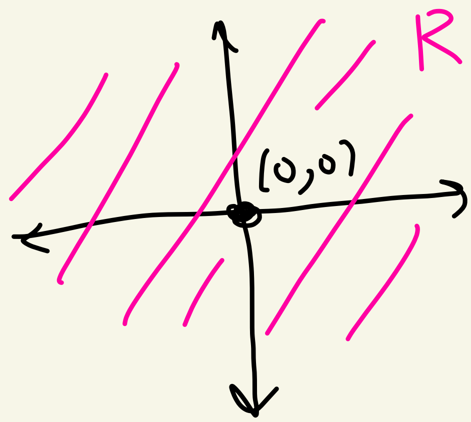
③ $y' = 2xy$

$f(x,y) = 2xy$

$\frac{\partial f}{\partial y} = 2x$

Both f and $\frac{\partial f}{\partial y}$ are continuous on the entire xy-plane. Let R be the xy-plane. Since $(0,0)$ is in R by Picard's theorem there exists a unique solution to

$$y' = 2xy$$
$$y(0) = 0$$



on some interval I containing $x = 0$.

④ Multiply $x^2y' + xy = 1$ by $\frac{1}{x^2}$ to get

$$y' + \frac{1}{x}y = \frac{1}{x^2}. \quad (*)$$

Let $A(x) = \int \frac{1}{x}\,dx = \ln|x| = \ln(x)$

$\boxed{x>0}$

Multiply both sides of $(*)$ by

$$e^{A(x)} = e^{\ln(x)} = x \quad \text{to get}$$

$$xy' + y = \frac{1}{x}$$

This gives

$$(xy)' = \frac{1}{x}$$

So,

$$xy = \int \frac{1}{x}\,dx$$

So,

$$xy = \ln|x| + C$$

$\boxed{\begin{array}{c}\ln|x| = \ln(x) \\ \text{since } x>0\end{array}}$

Thus,

$$\boxed{y = \frac{1}{x}\ln(x) + \frac{C}{x}}$$

5 See HW 4 #1(d)

6 See HW 5 #1(b)